Experimental verification of the synchronization condition for chaotic external cavity diode lasers

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The synchronization condition obtained numerically by Ahlers, Parlitz, and Lauterborn [Phys. Rev. E **58**, 7208 (1998)] is verified experimentally.

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Synchronization phenomena are of fundamental importance for various physical, chemical, and biological systems. Synchronization of chaos has aroused much interest in light of its potential applications in secure communications. Several papers have shown that such synchronization may be achieved in electronic oscillator circuits [1–3]. Gaponov-Grekhov, Rabinovich, and Starobinets have developed a theory of synchronization of coupled, chaotic, nonlinear oscillators in the context of turbulence of fluids [4]. Synchronization of chaos is understood to be the perfect coincidence of the chaotic dynamics of two coupled chaotic systems. Synchronized chaos has been observed experimentally in Nd:YAG (yttrium aluminum garnet) [5], CO_2 [6], and NH_3 lasers [7]. Various synchronization algorithms have been proposed in the past [8] with quantitative measures of Lyapunov exponents [1,9] and with chaotic shift keying [10]. A recent theoretical study by Spencer *et al.* [11] led to the modeling of optical synchronization of chaotic external cavity vertical cavity surface emitting lasers (VCSEL's) and its dependence on the coupling coefficient between master and slave lasers, followed by experimental demonstration in chaotic external cavity diode lasers [12]. Synchronization depends on various parameters involved, viz., the operating conditions of the lasers, feedback levels, and coupling between the two systems. Recently, Ahlers, Parlitz, and Lauterborn [13] derived a condition for synchronization for external cavity diode lasers, which relates the feedback parameters of individual lasers and their coupling coefficient. In this paper we experimentally verify this relation.

A general form for this condition is obtained from consideration of the systems of Lang-Kobayashi equations for the real electric field amplitude $E(t)$, slowly varying phase $\phi(t)$, and carrier number $n(t)$ for the master laser (with subscript m),

$$\frac{dE_m}{dt}=\tfrac{1}{2}\,Gn_mE_m+\kappa_mE_m(t-\tau)\cos[\omega_0\tau+\phi_m(t)-\phi_m(t-\tau)],$$

$$\frac{d\phi_m}{dt}=\tfrac{1}{2}\,\alpha Gn_m-\kappa_m\frac{E_m(t-\tau)}{E_m(t)}\sin[\omega_0\tau+\phi_m(t)-\phi_m(t-\tau)],$$

$$\frac{dn_m}{dt}=(\rho-1)I_{\text{th}}-\gamma n_m-(\Gamma+Gn_m)E_m^2, \tag{1}$$

and for the slave laser (with subscript s) in the case of one-way coupling [13,14],

$$\frac{dE_s}{dt}=\tfrac{1}{2}\,Gn_sE_s+\kappa_sE_s(t-\tau)\cos[\omega_0\tau+\phi_s(t)-\phi_s(t-\tau)]+\sigma E_m(t-\tau_c)\cos[\omega_0\tau_c+\phi_s(t)-\phi_m(t-\tau_c)],$$

$$\frac{d\phi_s}{dt}=\tfrac{1}{2}\,\alpha Gn_s-\kappa_s\frac{E_s(t-\tau)}{E_s(t)}\sin[\omega_0\tau+\phi_s(t)-\phi_s(t-\tau)]-\sigma\frac{E_m(t-\tau_c)}{E_s(t)}\sin[\omega_0\tau_c+\phi_s(t)-\phi_m(t-\tau_c)], \tag{2}$$

$$\frac{dn_s}{dt}=(\rho-1)I_{\text{th}}-\gamma n_s-(\Gamma+Gn_s)E_s^2.$$

Here G is the differential optical gain, τ the master laser's external cavity round-trip time, α the linewidth enhancement factor, γ the carrier density rate, Γ the cavity decay rate, ρ the pump current relative to the threshold value J_{th} of the free running laser, ω_0 the angular frequency of the free running laser, κ the feedback rate, expressed as the photon number per second (e.g., the typical value of κ used in Ref. [13] is $10^{10}\,\text{s}^{-1}$), σ the coupling strength between the master and slave lasers, and τ_c the light propagation time from the front facet of the master laser to the front facet of the slave laser.

It is clear that the synchronization occurs with some delay (lag synchronization). The delay time is introduced because

$$\Delta t=\tau_c-\tau.$$

It can easily be seen from comparison of Eqs. (1) and (2) that such synchronization between the master and slave lasers,

$$E_m(t)=E_s(t-\Delta t),$$

$$n_m(t)=n_s(t-\Delta t), \tag{3}$$

$$\phi_m(t)=\phi_s(t-\Delta t)-\omega_0\Delta t(\text{mod}\ 2\pi),$$

takes place if the following condition is satisfied:

$$\kappa_m=\kappa_s+\sigma\frac{\cos(\omega_0\tau_c)}{\cos(\omega_0\tau)}. \tag{4}$$

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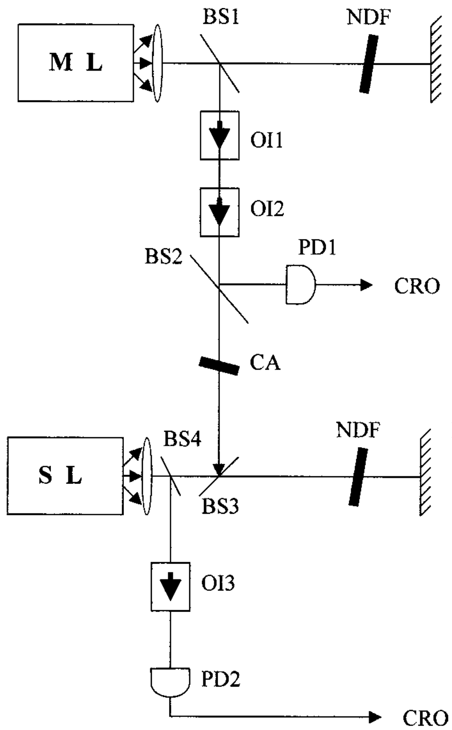

FIG. 1. Schematic diagram of the experimental arrangement: ML, master laser; SL, slave laser; BS1–BS4, beam splitters; PD1,PD2, photodetectors; OI1–OI3, optical isolators; M1,M2, mirrors; NDF's, neutral density filters; CA, coupling attenuator; CRO, digital oscilloscope.

Under most experimental conditions $\cos(\omega_0\tau_c)\approx\cos(\omega_0\tau)$ and hence

$$\kappa_m=\kappa_s+\sigma. \tag{5}$$

Although the condition in (5) was derived under the assumption of identical lasers, the experimental results confirm its validity for practical lasers which of course are not identical except to the extent that they are of the same make and model. The above condition would not be expected to hold for strongly dissimilar lasers. However, in that case it would not be expected that synchronization can be obtained.

The condition in (5) was proposed by Ahlers, Parlitz, and Lauterborn [13] based on the concept developed by Kocarev and Parlitz [15]. First each of the two chaotic systems connected by some driving signal can be rewritten as a system driven by a common (for both systems) signal which is some function of the driving signal between the chaotic systems [16]. Next, one makes the right-hand side of the equations describing the system dynamics identical on the synchronization manifold. The condition (4) in fact is obtained from a comparison between the coefficients of the electric fields (or the phase) for the master and slave lasers when Eq. (3) is fulfilled. This active-passive decomposition approach [15] to chaos synchronization is a generalization of the seminal Pecora-Carroll method [1]. Recently, this approach was applied numerically by Parlitz and collaborators to the case of chaos synchronization between two laser systems.

The experimental arrangement is shown schematically in Fig. 1. Use has been made of two commercial semiconductor lasers. These are single-mode Fabry-Perot (FP) lasers emitting at 830 nm, with a linewidth of 200 MHz (Access Pacific model APL 830-40) and a threshold current $J_{\rm th}$ of 42 mA. The side mode suppression ratio is -20 dB. These lasers are driven by ultralow noise current sources (ILX-Lightwave, LDX-3620) and are temperature controlled by thermoelectric controllers to a precision of 0.01 K (ILX-Lightwave LDT-5412). The laser output is collimated using an antireflection coated laser diode objective (Newport FLA11). Both the lasers are subjected to an optical feedback from external mirrors ($M1$ and $M2$) and the feedback strength is controlled using continuously varying neutral density filters (NDF1 and NDF2). The cavity length is 76 cm in both the cases. The optical isolators (OFR-IO-5-NIR-HP) ensure that the lasers are free from back reflection and the typical isolation is -41 dB. Isolators OI1 and OI2 ensure that the master is isolated from the slave laser. The coupling attenuator (CA) enables the percentage of master power fed into the slave laser to be controlled. PD1 and PD2 are two identical fast photodetectors (New Focus model 1621) with a response time of 2 ns. The output of the master laser is coupled to the photodetector PD1 by the beam splitters BS1 and BS2. Beam splitter BS3 acts as a coupling element between the master and slave. Beam splitter BS4 couples the slave output to the photodetector PD2. Photodetector outputs are stored in a digital storage oscilloscope (LeCroy LC564A).

Both the master and slave lasers are rendered chaotic by appropriate amounts of feedback. The effective external cavity reflectivities for the master (R_m) and slave (R_s) lasers are 1.98×10^{-2} and 2.52×10^{-3}, respectively. 10% of the master laser output is fed to the slave laser through the coupling beam splitters. The slave laser output is displayed on the oscilloscope, incorporating the correction arising due to the time delay. Synchronization is achieved between the two chaotic external cavity diode lasers [12] and the synchronization plot can be seen on the oscilloscope screen in X-Y display mode. The current level at which the synchronization

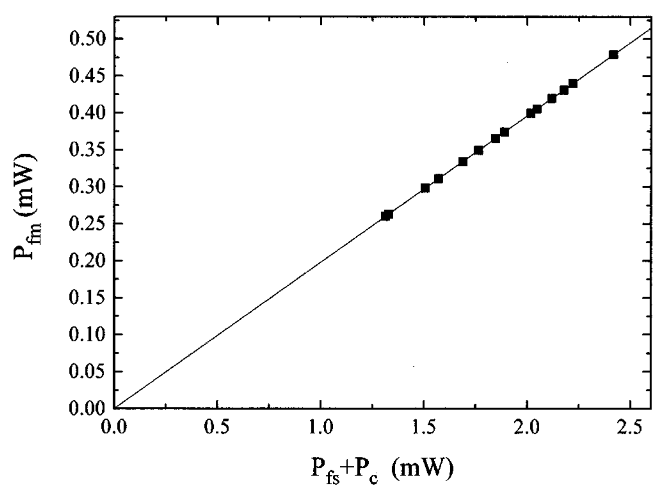

FIG. 2. P_{fm} vs $(P_{fs}+P_c)$ under conditions of synchronization. P_{fm} is the feedback power to the master laser, P_{fs} is the feedback power to the slave laser, and P_c is the master power coupled to the slave laser. The size of the squares appearing in the figure corresponds to a power variation of about 0.01 mW. Maintenance of the synchronization requires current fluctuations less than 0.25 mA, which would produce a change of approximately 0.01 mW in the feedback power.

is achieved is noted to a precision of 0.01 mA. The master laser operating current is varied to another value, which leads to total degradation of synchronization. However, the slave laser drive current is varied to regain synchronization. This procedure is repeated several times. The lasers are operated from $1.5J_{\rm th}$ to $2.0J_{\rm th}$. The output power levels of the master and slave lasers at these current levels are measured. This allows us to calculate the feedback power to the master (P_{fm}) and slave (P_{fs}) lasers and the master power coupled to the slave laser (P_c). The quantities P_{fm}, P_{fs}, and P_c are proportional to κ_m, κ_s, and σ respectively. A graph is plotted of P_{fm} versus ($P_{fs}+P_c$) and is shown in Fig. 2. This shows a linear dependence between P_{fm} and ($P_{fs}+P_c$). The linearity provides a qualitative verification of Eq. (5).

Once synchronization is achieved for a particular master laser current, the lasers remains synchronized for a change in the slave laser current of ± 0.25 mA. A change in the operating current of 0.25 mA corresponds to a change in the feedback power of approximately 0.01 mW, which is represented by the size of the squares appearing in the figure. Thus no noticeable change in the output power will occur while synchronization is maintained and therefore the points in the figure fall exactly on the drawn straight line.

In conclusion, the synchronization condition obtained by Ahlers, Parlitz, and Lauterborn [13] is verified experimentally.

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